

NUMERICAL PREDICTION OF UNDRAINED RESPONSE OF PLATE ANCHORS UNDER COMBINED TRANSLATION AND TORSION

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ABSTRACT

The undrained pure translational and torsional capacity of anchors and the plate response under eccentric translational loading is investigated in this paper using three-dimensional finite element (3D-FE) analysis. Plastic limit analysis is adopted to establish a benchmark solution for ultimate translational resistance with satisfactory agreement with the FE values which confirms the developed numerical model. Although plate thickness has a marked impact on the maximum shear and torsion resistance, the shape of failure envelope is minimally affected by thickness. A simple three-degree-of-freedom interaction equation is curve-fitted to FE failure datapoints. Representative interaction relationships are introduced for square and rectangular plates.

KEY WORDS: plate anchor; torsional capacity; translational (shear) resistance; plastic limit analysis; finite element analysis; failure envelope.

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1- Introduction

The offshore industry is expanding farther into very deep water, leading to a number of economical and technical challenges. Floating structures anchored to the seabed using catenary and taut-wire mooring systems are generally more technically feasible and cost effective than gravity based platforms in these deep water environments. Anchorage for floating systems can include piles, caissons and various types of plate anchors. This paper addresses performance of the latter alternative. Under normal operating conditions, plate anchors are mainly subjected to pull-out loading in an intended plane of loading, for example, the X-Z plane in Fig. 1(a). However, extreme storm events may cause partial failure with the loss of one or several mooring lines resulting in substantial changes in the orientation of the intact lines, as well as to the applied resultant force [Fig. 1(b)]. Under these conditions the anchor must resist a general system of loads, with some components of load acting outside the plane of intended loading.

A full description of behavior of an anchor plate in a partially failed mooring system requires adequate understanding of undrained response of anchor plates subjected to general loading conditions involving six degrees of freedom. This entails characterizing anchor behavior for six components of uniaxial loading (3 translational and 3 rotational) as well as the interaction effects amongst all possible load combinations. Much of the research in this area to date has focused on in-plane loading conditions involving forces in the X and Z directions and moments about the Y-axis (Fig. 1) (O'Neill et al. 2003, Murff et al. 2005). This paper investigates anchor behavior for loading in the X-Y plane in Fig. 1, which will be termed 'shear-torsional' loading. The loading comprises 'shear' forces H_x and H_y and a torsion T ; i.e., a moment acting about the Z-axis. All loads act within the plane of the plate. The study presented herein does not attempt to fully characterize anchor behavior for general 6 degree of freedom loading conditions. Rather, it investigates a critical component for a more general model to be developed in the future.

The plate anchors considered in this paper are idealized as rectangular plates with specified aspect ratios (W/L) and thickness t (Fig. 1). Some plate anchors, such as suction embedded plate anchors (SEPLAs) match this idealized geometry to a reasonable degree. By contrast, drag embedded plate anchors usually have relatively complex geometric configurations to facilitate embedment. The findings of this paper can provide useful insights into the behavior of drag anchors, but the inherent limitations of the analyses arising from the simplified geometry must be carefully considered. O'Neill et al. (2003) used plane strain finite element (2D-FE) analysis to investigate the behavior of rectangular and wedge-shaped strip anchors subjected to combined in-plane translational, vertical, and rotational loading. They also developed plastic upper bound solutions to evaluate pure parallel, normal, and rotational capacity factors for strip plates to validate the FE results. Using two-dimensional finite element (2D-FE) analysis, they produced failure loci and plastic potentials from considerations of fluke-soil interaction and then developed a design method to predict drag anchor trajectory during embedment. Murff et al. (2005) used 2D-FE to develop a failure locus for the ratio of plate length to thickness (L/t) of 7 and 20 with different plate roughness. Yang et al. (2010) employed 3D-FE, to study the behavior of infinitely thin plate anchors subjected to loading in all six degrees of freedom. They also introduced a plasticity solution for failure envelope of a plate under combined translation-torsion and developed formulas to estimate pure sliding and torsional bearing capacity. This plasticity solution predicts the behavior of an infinitely thin plate, but provided only an approximate solution for a plate with finite thickness. Nouri (2013) modified the plastic limit solution originally developed by Yang et al. (2010) to predict more accurately the torsional collapse load and failure envelopes of a plate anchor with finite thickness. Researchers have also conducted 3D-FE analysis on other types of offshore foundations under general loading condition, such as for mudmats (Feng et al. 2014) or OMNI-max anchor (Wei et al. 2015). In the current study, we use 3D-FE analysis to evaluate the translational (parallel to plate) and torsional ultimate bearing capacity of plates, as well as the interaction of these two modes of loading.

Results of the FE analysis are also used to validate analytical solutions to predict bearing capacities and to confirm whether the postulated failure mechanism for the analytical solution is correct.

2- Material Properties for Numerical Model

The commercial software *ABAQUS* (HKS, 2008) is used in this study. The plate is assumed to be deeply embedded, with localized plastic flow forming around the plate anchor and not extending to the surface, resulting in capacity factors that are not affected by overburden and soil weight (Song et al. 2008; Wang et al. 2010). Thus, the soil is assumed to be a weightless material without loss of accuracy. Since the size of the plastic zone around the plate anchor at failure is smaller for shear and torsion than for uplift and rotation, the assumption of deep embedment is valid even for shallower embedment depths. This assumption also implies that the soil and plate are fully bonded. Thus, “no separation” is assumed as the normal (i.e. perpendicular to interface) contact behavior for the plate-soil interface in conjunction with “rough” for the tangential contact property to realistically simulate a fully bonded condition. The interaction between soil and plate is modeled using surface to surface contact pairs in which the plate outer surface is chosen as a “master surface” and the soil surface in contact with the plate as a “slave surface”. The active degrees of freedom of the nodes on the slave surface are constrained to the master surface nodes through relationships which define the tangential and normal interaction at the nodes of these two surfaces.

Clay under undrained conditions is modeled as a linear elastic perfectly plastic material, with yielding determined by the Von Mises failure criterion with undrained shear strength, s_u . The Young’s modulus of the soil, E , is given by a modulus ratio of $E/s_u = 500$, and Poisson’s ratio is taken as 0.49 to simulate no volume change for undrained clay in total stress analysis. The ultimate capacity of the plate is not affected by the pre-failure elastic behavior of the soil (Chen and Liu 1990). In addition, in studies on plate uplift bearing capacity it was observed that as the soil rigidity (E/s_u) increases, the anchor displacement required to mobilize the maximum capacity reduces, while the maximum capacity does not change (Song et al. 2008; Wang et al. 2010). There-

fore, the Young's modulus of the soil is assumed constant in these studies. Since the analysis of the structural behavior of the anchor is not within the scope of this work, the plate is modeled as a rigid body with Young's modulus 10^{10} times that of the soil, and Poisson's ratio of 0.30. The analysis is conducted using small strain displacement control, since the ultimate translational and torsional capacities are developed within an anchor movement of 0.1 times the anchor length. A reference node is defined in the center of the plate and prescribed displacements are applied at the reference point to perform displacement controlled analysis. Standard boundary conditions are also applied to the model: the base is fixed in all directions, while the vertical boundaries are fixed in the lateral direction and free to move vertically. In all the models with a plane of symmetry, the nodes on the plane of symmetry are only allowed to move in the plane.

3- Challenges of the Numerical Modeling

Unrealistic stress distributions arise from the displacement discontinuities that take place at the edges of a rigid structure penetrating a softer material (Van Langen and Vermeer 1991) resulting in higher overall resistance. The unfavorable effect on the numerical results is more severe for buried structures and materials with no volume change. There are mainly two approaches to overcome such loss of numerical accuracy:

1- Mesh refinement

2- Contact pairs along the corners of the plate

As plane strain 2D analyses take much less time and fewer memory resources, they are a reliable and fast way to evaluate possible solutions for this problem. 2D-FE analyses can also be adopted to develop an optimal mesh size and to verify properties and length of the contact pairs in soil along the plate corners through sensitivity analysis. The plate anchor is idealized as a strip with length L and thickness t in 2D space.

3-1- Mesh refinement around the sharp edges

As the detrimental effect of the stress concentration is more pronounced in thinner plates, a plate with thickness of $t = L/20$, is considered for the preliminary 2D analysis to calculate the ultimate translational (parallel to plate) bearing capacity of the strip plate. Several 2D models with coarse, fine, and very fine mesh with first order 4-node plane strain quadrilateral hybrid elements, CPE4H, are created. All the soil elements are fully integrated hybrid type to accommodate the constant volume constraint in undrained conditions. The whole model dimensions are $9L \times 9L$, in which L is the length of the plate. The model is divided into several zones with different mesh densities to minimize the degrees of freedom of the model and avoid unnecessary computational effort. The finest zone extending to $t = L/20$ around the plate consists of square elements with dimension of $L/80$, $L/160$, $L/320$ for the coarse, fine, and very fine meshes, respectively. A fine mesh with second order hybrid square elements (CPE8H) with dimension of $L/160 \times L/160$ is also created to assess the performance of higher order elements in predicting ultimate bearing capacity. Only a small difference can be observed in the maximum parallel bearing capacity of the plate for the fine and very fine meshes in Fig. 2. The results from the fine mesh (elements of $L/160 \times L/160$) with second order elements match exactly the values of parallel bearing capacity from the very fine mesh with first order elements. Refining the mesh improves the results and overcomes the issues of stress concentration to some extent, but the bearing capacity values do not change significantly after some degree of mesh refinement. The ultimate shear capacities calculated using the very fine mesh or higher order elements are still 13% higher than the value obtained from the upper bound plastic limit analysis. In addition, a 3D fine mesh with elements of size $L/160 \times L/160$ would require significant computational effort, making the numerical analysis practically impossible to conduct. Thus, for a reasonable computational effort it seems that excessive mesh refinement fails to provide sufficiently accurate FE results for shear bearing capacity of plate anchors deeply embedded in undrained clay.

3-2- Contact interfaces in soil along the corners of the plate

An alternative solution to enhance the stress distribution on the plate edges is using contact interfaces near the corners of the plate. Rowe and Davis (1977) introduced potential rupture lines near the edge of an anchor to ensure accurate modeling of the singularity at the anchor tip. This approach attempted to overcome the inhibition of free plastic flow inherent in the usual stiffness formulation of the FE method by permitting the formation of velocity discontinuities in the regions of high stress and velocity gradient near the tip of the anchor plate (Rowe and Davis, 1977). Van Langen and Vermeer (1991) introduced zero-thickness interface elements to enhance the realistic stress non-uniformity on surfaces with singular points.

Using the same general concept, and taking advantage of *ABAQUS* capabilities in defining various types of contact relationships, this study adopts interfaces placed vertically and horizontally in the soil in the vicinity of plate corners to accommodate for vertical and horizontal displacement discontinuities. Each interface is defined as a couple of master and slave surfaces in the soil that can slide independently from each other according to a surface to surface contact relationship (contact pairs). A coefficient of friction ($\mu = 0.001$) and a maximum contact shear stress equal to the undrained shear strength of the soil ($\tau_{max} = s_u$) are adopted to simulate frictional behavior. No separation between the master and slave surface nodes is also assumed in the direction normal to the interface.

As shown by O'Neill et al. (2003) and confirmed through the 2D-FE analysis, in-plane horizontal movement of the rigid plate creates a plastic zone in the soil around the plate that extends for a distance from the plate approximately equal to plate thickness, t . Contours of deformation in the x direction for a plate under pure sliding in the 2D-FE analysis shown in Fig. 3 also confirm the extents of this plastic zone. The 3D-FE analysis also shows a similar zone of thickness t to be affected by the torsional rotation (Fig. 4). The finest zone wrapping around the plate contributes the most to the number of elements, i.e. the computational effort in the model.

Thus, this region extends for a distance from the plate equal to the plate thickness, t , to minimize the number of elements. The length of the contact is also adopted equal to plate thickness. The final 2D mesh and the contacts placed horizontally and vertically in the vicinity of the plate corners are shown in Fig. 5. The final mesh is divided into six different zones with element size decreasing as the zones become closer to the plate. Mesh tie constraints are used to constrain each active degree of freedom of the slave node on the finer zone surface to the master node on the coarser zone surface. FE results for the ultimate parallel bearing capacity are compared with the plastic limit analysis (PLA) value presented by O'Neill et al. (2003). For a plate with $t = L/20$ and $\mu=0.001$, the FE-derived value of the plane strain shear bearing capacity factor is 2.77, which exceeds the PLA result, 2.75, by less than 1%. The effect of the interface coefficient of friction, μ , on the ultimate shear capacity results is evaluated for coefficient values ranging from 0.001 to 1000 (Fig. 6). For higher values of friction coefficient the ultimate capacity increase to about 2.85 and remains constant for coefficient values larger than one ($\mu > 1$), which shows a minor impact of the friction coefficient on the results. Thus, the friction coefficient of $\mu=0.001$ is adopted for the interface in all the following analysis. FE results for the ultimate parallel bearing capacity also indicate that the mesh is adequately fine, contact length and properties are reasonable and the equivalent 3D mesh could give satisfactory results.

4- 3D finite element analysis

The 3D mesh is created with the same approach used for the 2D mesh (Fig. 7). The 3D model is created for anchors idealized as square and rectangular ($W/L = 2.0$) plates with thickness $t = L/20$, $L/14$, $L/10$, and $L/7$. The model dimensions are $9L \times 9L \times 5L$ and $9L \times 12L \times 5L$ for the square and rectangular plates, respectively. The 3D mesh consists of 8-node full integration 3D hybrid first order elements, C3D8H. Similarly to the 2D analyses, the Von Mises constitutive model with the same elastic-plastic properties is used for the soil. The contact definition of the 2D model is used at the plate corners in the soil and plate-soil interface. The length of the contact and the depth of the fine

mesh zone around the plate are equal to the plate thickness in all models (Fig. 4). Taking advantage of the symmetric nature of shear-torsion loading of the plate anchor, only half of the FE mesh is simulated. Standard boundary conditions are applied to the model boundaries and plane of symmetry.

4-1- Ultimate load capacity

In-plane translational (parallel to plate) displacements are applied in the direction parallel to the smaller (L) and larger (W) sides of the plate (x and y directions respectively) and torsion is prescribed as the rotation of the plate about the z axis. The FE results for the pure shear and pure torsional loading are reported in Tables 1 and 2, respectively, for square and rectangular ($W/L = 2$) plates, in their normalized forms, $N_{sx,max}$, $N_{sy,max}$, and $N_{t,max}$, where:

$$N_{sx,max} = \frac{H_{x,max}}{LW s_u} \quad (1)$$

$$N_{sy,max} = \frac{H_{y,max}}{LW s_u} \quad (2)$$

$$N_{t,max} = \frac{T_{max}}{L^2 W s_u} \quad (3)$$

where H_x , and H_y , are the horizontal (shear) reaction force in x and y directions and T is the torsional reaction moment of the rigid anchor plate about the z axis.

The FE ultimate shear resistance is compared with benchmark solutions: (1) the resistance factor for an infinitely thin plate, which provides the minimum values of ultimate shear resistance ($N_{sx,max}=N_{sy,max}=2$ for $H_{x,max}=H_{y,max}=2s_uWL$); (2) A PLA formulation, developed by Yang et al. (2010) to evaluate the plate thickness impact on the ultimate shear capacity (Table 1):

$$N_{sx,max} = \frac{H_{x,max}}{s_u L W} = 2\alpha + 2 \left[\alpha \frac{L}{W} + N_e \right] \frac{t}{L} \quad (1)$$

$$N_{sy,max} = \frac{H_{y,max}}{s_u L W} = 2\alpha + 2 \left[\alpha + N_e \frac{L}{W} \right] \frac{t}{L} \quad (2)$$

where N_e is the simple plane strain bearing capacity factor, equal to 7.5 (O'Neill et al. 2003). The adhesion factor, α , for the fully bonded condition is assumed equal to unity. The above expression for pure x -shear capacity factor, $N_{sx,max}$, for a strip plate ($L/W \approx 0$) reduces to $N_{s,max}=2(1+N_e t/L)$ as proposed by O'Neill et al. (2003) for ultimate parallel plate capacity in 2D plane strain condition. Whereas the PLA analyses under-predict FE shear capacity factors for plate thickness of $t = L/20$, and $L/14$, they over-predict FE capacity factors for $t = L/10$ and $L/7$ for both square and rectangular plates. This could be attributed to the fact that the shear bearing capacity of thicker plates is less sensitive to stress concentration on plate corners because the ultimate shear bearing capacity of thick plates is larger than that of thinner ones. Overall, the 1-3% difference between PLA and FE results for ultimate shear capacity is minimal. The thickness has a marked impact on the plate shear resistance, resulting in an approximately 40 to 120% increase for square plates of $t=L/20$ and $L/7$ respectively, compared to the $t=0$ plate. The typical normalized load-displacement responses for square and regular plates under in-plane y -shear are also depicted in Fig. 8. Table 2 summarizes the FE derived maximum torsional resistance. Torsion-rotation plots are also included in Fig. 9. Yang et al. (2010) also proposed a PLA solution for pure torsion, which tends to over-predict resistance for plates of finite thickness (Nouri 2013), but their formulation for an infinitely thin plate ($N_{t,max0}$) provides a lower bound value for ultimate torsional resistance:

$$N_{t,max0} = T_{max0} / (s_u W L^2) = \alpha \frac{(W/L)^2}{6} \left\{ \frac{\sin \theta_o}{\cos^2 \theta_o} + \ln \left[\tan \left(\frac{\pi}{4} + \frac{\theta_o}{2} \right) \right] \right\} + \alpha \frac{L/W}{6} \left\{ \frac{\cos \theta_o}{\sin^2 \theta_o} - \ln \left[\tan \left(\frac{\theta_o}{2} \right) \right] \right\} \quad (3)$$

where $\theta_o = \tan^{-1}(L/W)$. For square and rectangular plates this analytical solution yields 0.76 and 1.19, respectively. Torsional resistance also increases markedly with plate thickness by approximately 40 to 90% for $t=L/20$ and $L/7$, respectively, versus the zero thickness plate.

4-2- Failure envelopes

In order to construct the interaction curve or failure locus, two types of displacement control methods are used: the swipe test and the probe test. In the swipe test approach failure locus is obtained by a single analysis in two separate steps. In the swipe test the plate is firstly displaced in the direction of the degree of freedom (DoF) under examination from zero until the ultimate capacity in that direction is reached. In the second step, the displacement is imposed in the second DoF until the ultimate capacity in the new direction is fully developed. In the finite element analysis once the ultimate force is reached in the first step, there will be no more increase in the reaction force along that DoF, thus no further expansion in overall plate failure locus as the movement of the plate progresses. The disadvantage of this method is that, due to the elasto-plastic “yielding” occurring within the “failure” locus, the swipe test tracks a load path marginally inside the actual overall plate failure envelope (Bransby and Randolph 1998).

In the probe test or fixed displacement method, a single point on the failure envelope is identified for each fixed ratio of the prescribed displacements. Therefore, the failure locus can be created by conducting a number of finite element analyses with different displacement ratios. The prescribed fixed displacement ratio gives rise to a load path beginning at the origin with an initial gradient based on the elastic stiffness of the material. The path gradient decreases as approaching failure envelope to follow the interaction curve and stops where there is no further increase in the forces developed in each intended degree of freedom (Bransby and Randolph 1998). This method gives an accurate trace of the failure envelope, but requires several FE analyses.

4-2-1- Combined in-plane shear and torsion

Fig. 10 shows examples of the swipe and probe failure envelopes for square and rectangular plates of $t = L/7$ subjected to in-plane y -translation and torsion ($N_{sy} - N_t$). The rest of the analyzed cases are not included for brevity. Note that $du_y/d\phi$ is the prescribed fixed displacement ratio for each probe path, defined as the ratio of plate displacement increment in the y direction, to the increment of the plate in-plane rotation. The probe curve exceeds the shear-torsion (i.e. shear prescribed first,

torsion next) swipe envelope. This difference increases significantly if a torsion-shear swipe test (i.e. torsion first, shear next) is carried out instead. In the torsion-shear swipe test, at the moment of full development of the torsional ultimate capacity, the soil elements in regions close to the middle of the plate and the plate sides are not completely plastic yet since the maximum displacements occur in the region next to the corners of the plate (See Fig. 4). Thus, the stress state of the soil elements in the center is farther from failure at the end of the first stage of pure torsion. In contrast, in the shear-torsion swipe test, all the soil surrounding the plate is fully plastic at the end of full mobilization of the shear resistance. All the swipe tests follow the shear-torsion sequence. As Fig. 10 suggests and as it was discussed earlier in the paper, the probe test approach offers more accurate values since the swipe envelope travels although marginally but still inside the probe curve. Therefore, probe test values are adopted and referred as FE results hereafter in the paper.

The failure envelope (e.g. Fig. 10) can be described by two basic components:

- 1- Maximum plate resistance under pure shear ($H_{x,max}$, $H_{y,max}$) and torsion (T_{max}) which quantifies the size of the envelope (discussed in previous section).
- 2- The shape of the failure envelope defined by the shear or torsional resistance normalized by the corresponding maximum value (e.g. $N_{sy}/N_{sy,max}$ - $N_t/N_{t,max}$ or $H_y/H_{y,max}$ - T/T_{max}).

Fig. 11 summarizes the FE probe results in the normalized space for all the studied cases. Interestingly, the plate thickness does not seem to affect the interaction behavior of the plate under one-way planar shear and torsion for the range of analyzed plate thicknesses, which is typical in practice. The interaction response is instead affected by plate aspect ratio and the direction of translation. Thus, a best fit for each aspect ratio and eccentric translation can be adopted as a representative failure envelope (Fig. 11).

The FE derived envelopes are compared with a PLA analytical solution, which was originally developed by Nouri et al. (2014) for surface foundations (i.e. zero embedment) under combined in-

plane translation and torsion. The formulation is slightly modified for a deeply embedded plate of zero thickness under the same loading combination. Adopting a PLA solution for zero plate thickness ($PLA_{t=0}$) has several advantages:

- It makes the PLA formulation easier and more manageable for excel spreadsheet applications and provides a straightforward benchmark solution for FE failure envelopes.
- Insensitivity of the normalized interaction response to plate thickness (Fig 11), makes the $PLA_{t=0}$ a simple yet reliable benchmark solution to evaluate the shape of failure envelope in normalized space for other plate thicknesses. Details of the proposed PLA formulation are summarized in Appendix A.

Fig. 12 compares the $PLA_{t=0}$ failure envelopes with FE values. Overall, the $PLA_{t=0}$ compares very well with the FE failure points for the square plate, slightly overestimates the resistance of rectangular plates under eccentric shear in x -direction, and underpredicts the bearing capacity for rectangular plates subjected to sheary-torsion. Besides providing a beneficial benchmark, $PLA_{t=0}$ potentially offers a simplified, yet effective, approach for finite thickness plates, if combined with the ultimate resistance factors for shear ($N_{sx,max}$ and $N_{sy,max}$) for $t \neq 0$ (Nouri et al. 2014).

Shear capacity reduction with eccentricity

Plotting the plate shear resistance versus the eccentricity offers an alternative and more intuitive portrayal of the plate shear capacity reduction with associated torsion. Fig. 13(a) shows examples of FE shear capacity reduction plots versus eccentricity for rectangular and square plates of $t = L/7$. The y -sliding capacity reduction plots for square and rectangular plates are equivalent to sheary-torsion failure envelopes in Fig. 10. Since the shear capacity in Fig. 13(a) is expressed as $N_s = H/WL_{su}$, the plots include both the size and shape elements for constructing a failure envelope. Similarly to Fig. 11, Fig. 13(b) summarizes all the FE results with a best fit load reduction curves, but in $(H/H_{max} - e/L)$ space. Fig. 13(b) indicates that plates of higher aspect ratio are more effective in resisting eccentric loads than square plates, irrespective of the orientation of the plate with

respect to shear. However, eccentric sliding in x -direction perpendicular to the longer side of the plate tends to reduce the shear capacity more significantly than eccentric y -sliding. The same trend was also observed in PLA capacity reduction curves for surface foundations (Nouri et al. 2014).

4-2-2- Co-planar shear (combined shear x -shear y)

FE analyses are also conducted to determine the interaction response of square and rectangular ($W/L=2$) plates of $t= L/20$ and $L/7$ under combined co-planar shear. Typical FE examples of swipe and probe envelopes for N_{sx} - N_{sy} are summarized in Fig. 14 for rectangular plate of $t = L/7$.

The shape of the co-planar normalized failure envelope in the $H_x/H_{x,max}$ - $H_y/H_{y,max}$ space (Fig. 15) is approximately the same for all plate thicknesses and aspect ratios. Therefore, a single curve for the FE data could be used to describe the normalized interaction response of plates with various thicknesses and geometries. The normalized co-planar shear ($H_x/H_{x,max}$ - $H_y/H_{y,max}$) failure envelope can be described using the following equation:

$$f(H_x, H_y) = \left(\frac{H_x}{H_{x,max}} \right)^{h_x} + \left(\frac{H_y}{H_{y,max}} \right)^{h_y} - 1 = 0 \quad (4)$$

Curve fitting yields interaction factors of $h_x=h_y=2.5$. For a zero thickness plate moving along any arbitrary in-plane direction, the plate shear resistance factor is always constant and equal to 2 regardless of the direction of plate translation. A constant resistance factor results in a circular locus with radius of 2 in the N_{sx} - N_{sy} space and a circle of radius 1 in $H_x/H_{x,max}$ - $H_y/H_{y,max}$ space (i.e. $h_x=h_y=2.0$ as shown in Fig. 15). This concept has also been validated through PLA by Nouri (2013). Comparison of circular ($t=0$) and FE shear x -shear y envelopes in Fig. 15 suggests that even a slight increase in plate thickness increases the plate shear resistance against the reduction induced by another shear force applied in the perpendicular direction. This increase tends to be insensitive to plate thickness at least for $t \leq L/7$ or changes in plate aspect ratio.

4-3- Interaction equation

Eq. (4) for concentric in-plane shear forces can be generalized for eccentric planar loads:

$$f(H_x, H_y, T) = \left[\left(\frac{H_x}{H_{x,max}} \right)^{hx} + \left(\frac{H_y}{H_{y,max}} \right)^{hy} \right]^h + \left(\frac{T}{T_{max}} \right)^{mz} - 1 = 0 \quad (5)$$

328 $f(H_x, H_y, T) = 0$ represents an ellipsoid in the $H_x/H_{x,max} - H_y/H_{y,max} - T/T_{max}$ normalized space.
 329 Since failure envelopes in the normalized space seem to be insensitive to plate thickness, the in-
 330 teraction factors of hx , hy , h , and mz are determined by curve fitting Eq. (5) to all FE best fit fail-
 331 ure envelopes for the square and rectangular plates in Figs. 11 and 15. Table 3 summarizes the
 332 calculated interaction factors.

333 5- CONCLUSIONS

334 The purpose of this study is to evaluate pure translational and torsional ultimate plate capacity, as
 335 well as to study the behavior of deeply embedded square and rectangular plates of finite thickness
 336 under combined co-planar shear and torsion using 2D and 3D finite element (FE) simulations.
 337 This study indicates the following:

- 338 1. Very fine meshes and higher order elements do not sufficiently eliminate stress concentration
 339 and overprediction of capacity for a deeply embedded plate in undrained condition. Applica-
 340 tion of contacts in the vicinity of the plate corners in soil improves bearing capacity esti-
 341 mates. The ultimate sliding and torsional capacities, as well as the interaction factors, are re-
 342 ported for a range of plate thicknesses and aspect ratios common in practice (Tables 1-2 and
 343 Figs. 8-9).
- 344 2. Plate shear resistance begins to reduce even at small eccentricity (e.g. 5% reduction for
 345 $e/L=0.1$ for square plate as shown in Fig. 13). Shear capacity reduces at a higher rate for
 346 larger eccentricity (e.g. 50% shear capacity reduction for $e=0.5L$ or eccentric shear force at
 347 the edge of square plate).
- 348 3. Reduction in shear resistance is affected slightly by the plate thickness but essentially de-
 349 creases with plate aspect ratio, since square plates are more vulnerable to reduction of sliding

capacity compared to rectangular plates (Fig. 13). In general, an eccentric shear load parallel to the shorter side of the plate induces greater capacity reduction in rectangular plates.

4. Thickness has a marked effect on a plate shear and torsional resistance. For perspective, the shear resistance factor equal to $N_s=2$ for a zero thickness plate, increases to about 4.4 for a square plate with $t=L/7$ (more than 100% increase). Similarly, the torsional resistance factor (N_t) increases by about 90% for square and rectangular plates with $t=L/7$.
5. The shape of the shear-torsion failure envelope in the normalized space ($H/H_{max} - T/T_{max}$) is insensitive to plate thickness (Fig. 11). This facilitates the application of analytical solutions developed for infinitely thin plates, $PLA_{t=0}$, (Nouri et al. 2014) to estimate the shape of the failure envelope, which shows a reasonable agreement with FE values (Fig. 13). As shown in Fig. 11, the shape of the shear-torsion failure envelope is affected by aspect ratio and the direction of the eccentric shear force.
6. The shape of the co-planar shear (shear_x-shear_y) failure envelope is generally insensitive to plate geometry and thickness, at least for the cases examined (Fig. 15).
7. A simple relationship is proposed to describe the failure envelope in the normalized space ($H_x/H_{x,max} - H_y/H_{y,max} - T/T_{max}$). Taking advantage of the insensitivity of the shape of the failure envelope to thickness, a representative envelope and the associated interaction factors are sufficient for practical design applications.

APPENDIX A. PLA virtual work formulation for zero thickness plate

For a horizontal load H applied at a distance e from the center of the plate (Fig A.1), equating external virtual work \dot{W} to internal energy dissipation yields to:

$$H = \dot{D} / (\rho + e) \dot{\beta} \quad (\text{A.1})$$

where $\dot{\beta}$ is a virtual angular velocity of the plate about a rotation center located a distance ρ from the center. The rate of internal energy dissipation \dot{D} is calculated by integrating the soil resistance times the local velocity over the plate area:

$$\dot{D} = 2s_u \dot{\beta} \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} \sqrt{(\rho + x)^2 + y^2} dx dy \quad (\text{A.2})$$

The dissipation term (\dot{D}) in the proposed solution is two times the \dot{D} for a surface foundation (Nouri et al. 2014), since the energy is dissipated along both top and bottom faces of the plate. This factor of 2 in the dissipation term is the only difference between this derivation and the PLA formulation for surface foundations (Nouri et al. 2014).

A least upper bound collapse load is obtained by minimizing H with respect to ρ , which leads to:

$$H = \frac{1}{\dot{\beta}} \frac{\partial \dot{D}}{\partial \rho} \quad (\text{A.3})$$

The collapse load (H) for the case of zero plate thickness becomes:

$$H = 2s_u \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} \frac{\rho_{opt} + x}{\sqrt{(\rho_{opt} + x)^2 + y^2}} dx dy \quad (\text{A.4})$$

where ρ_{opt} is the distance to the optimal center of rotation corresponding to a least upper bound.

Evaluation of the integral yields the following closed-form expression for H :

$$H = 2s_u \left[a_1^2 \ln \left| \frac{b_1 + W/2}{b_1 - W/2} \right| - a_2^2 \ln \left| \frac{b_2 + W/2}{b_2 - W/2} \right| + W(b_1 - b_2) \right]$$

$$a_1 = \rho_{opt} + L/2$$

$$a_2 = \rho_{opt} - L/2$$

$$b_1 = \sqrt{a_1^2 + W^2/4}$$

$$b_2 = \sqrt{a_2^2 + W^2/4}$$

(A.5)

The eccentricity, e , associated with any arbitrarily selected ρ is calculated from Eq. (A.1) with \dot{D} and H evaluated and substituted from Eqs. (A.2) and (A.4), respectively. Analytical integration of Eq. (A.2) is unwieldy; however, reduction to single integration is possible to facilitate simple design spreadsheet calculations:

$$\dot{D} = 2s_u \dot{\beta} \int_{-W/2}^{W/2} \left[(a_1 c_1 - a_2 c_2) + y^2 \ln \left| \frac{a_1 + c_1}{a_2 + c_2} \right| \right] dy$$

$$a_1 = \rho_{opt} + L/2$$

$$a_2 = \rho_{opt} - L/2$$

$$c_1 = \sqrt{a_1^2 + y^2}$$

$$c_2 = \sqrt{a_2^2 + y^2}$$

(A.6)

Eq. (A.6) can be integrated between the limits $-W/2$ to $W/2$ using classical numerical integration formulas.

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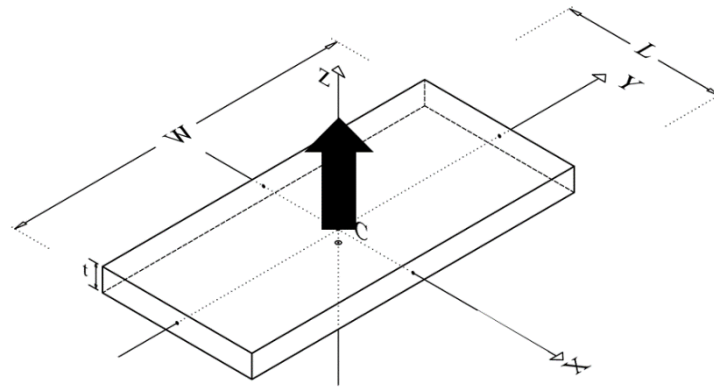
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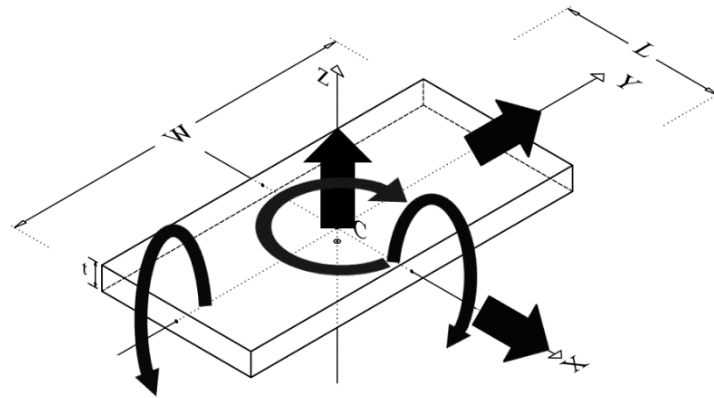
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(a)



(b)

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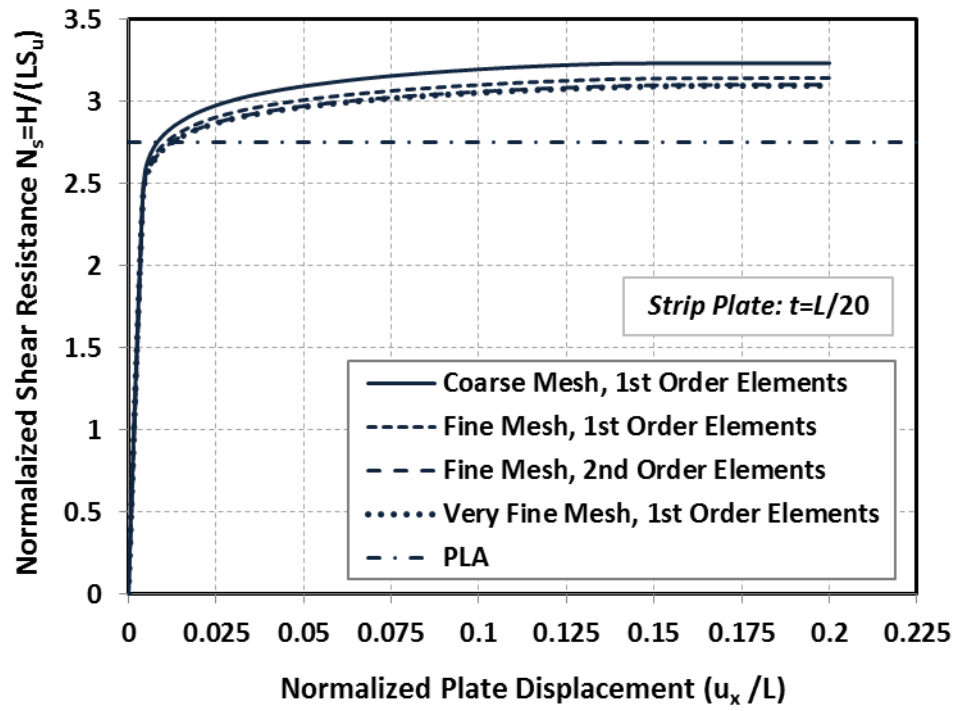


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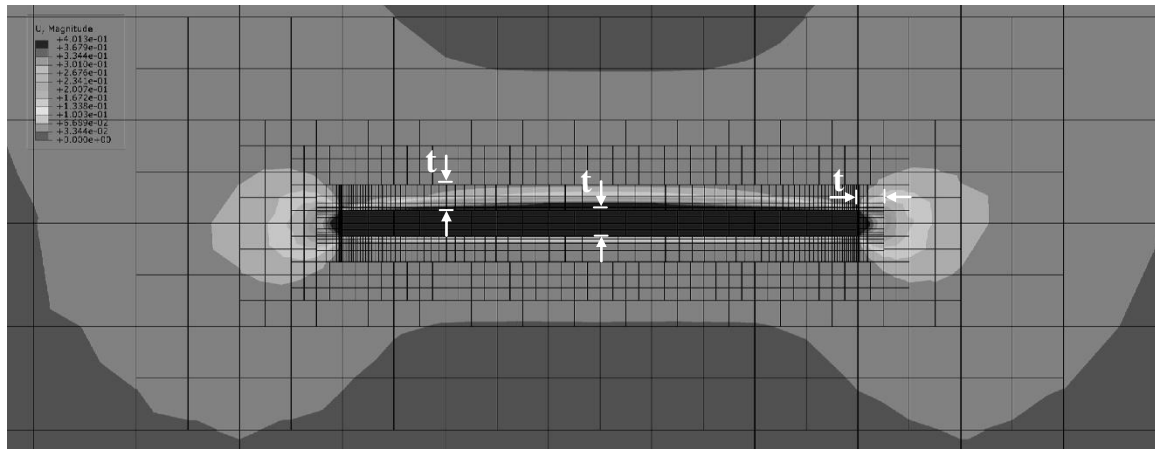
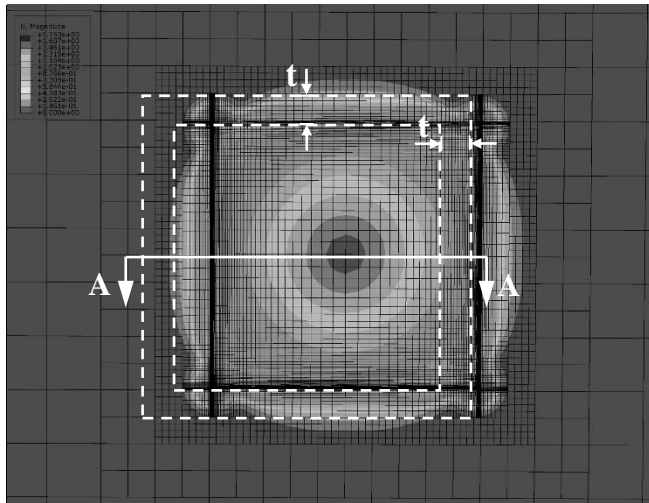
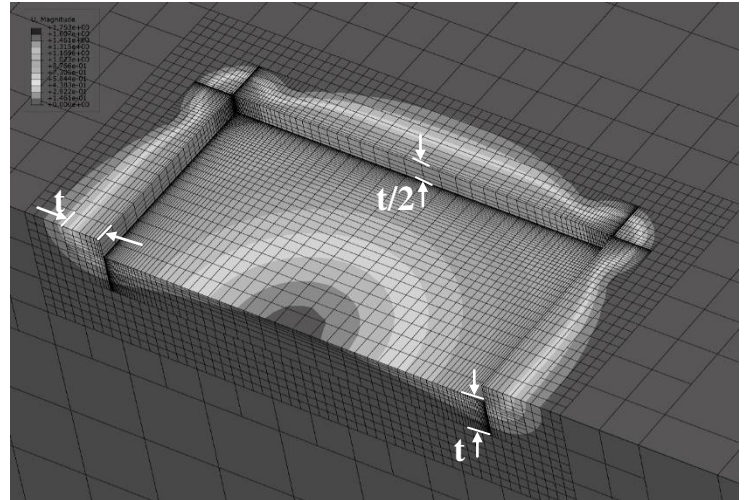


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(a)



(b)

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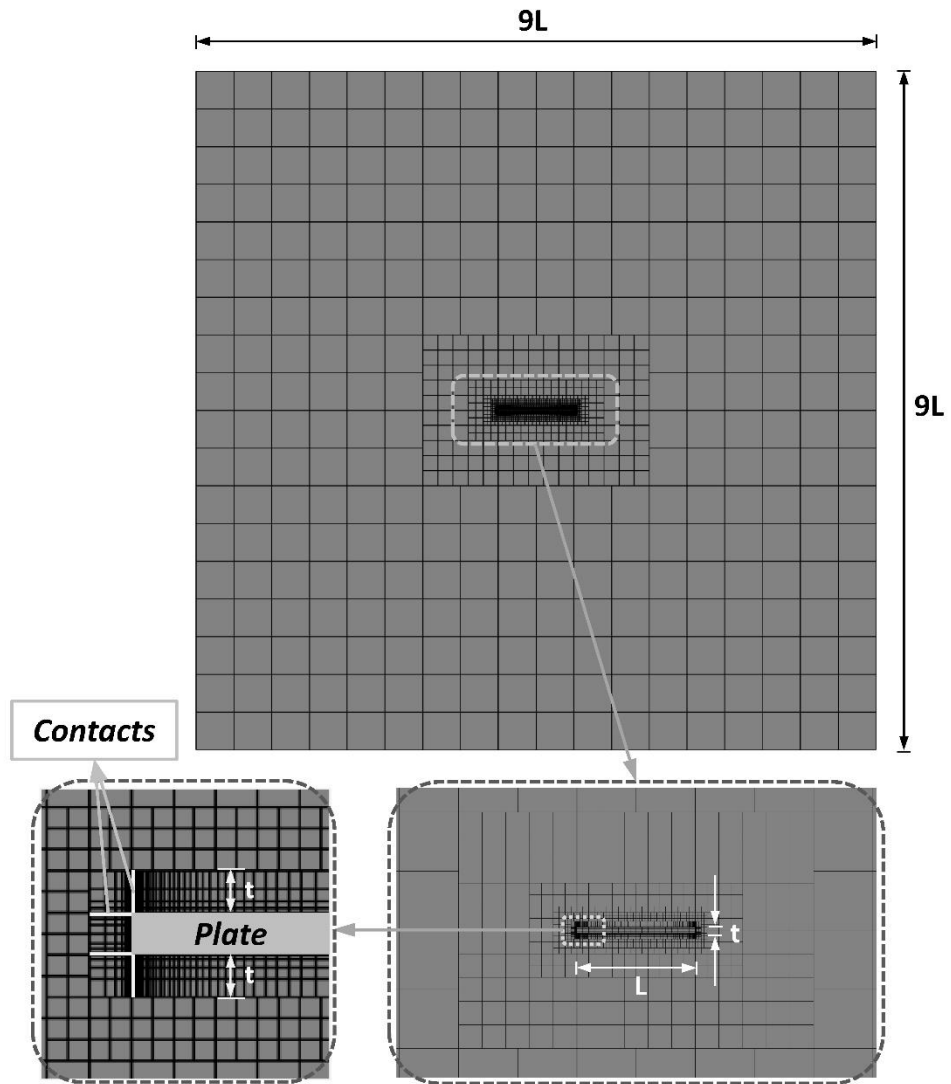


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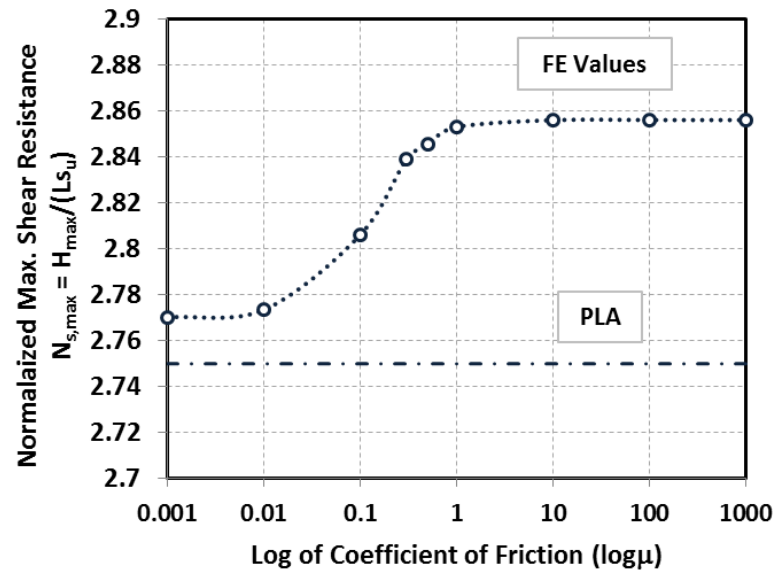


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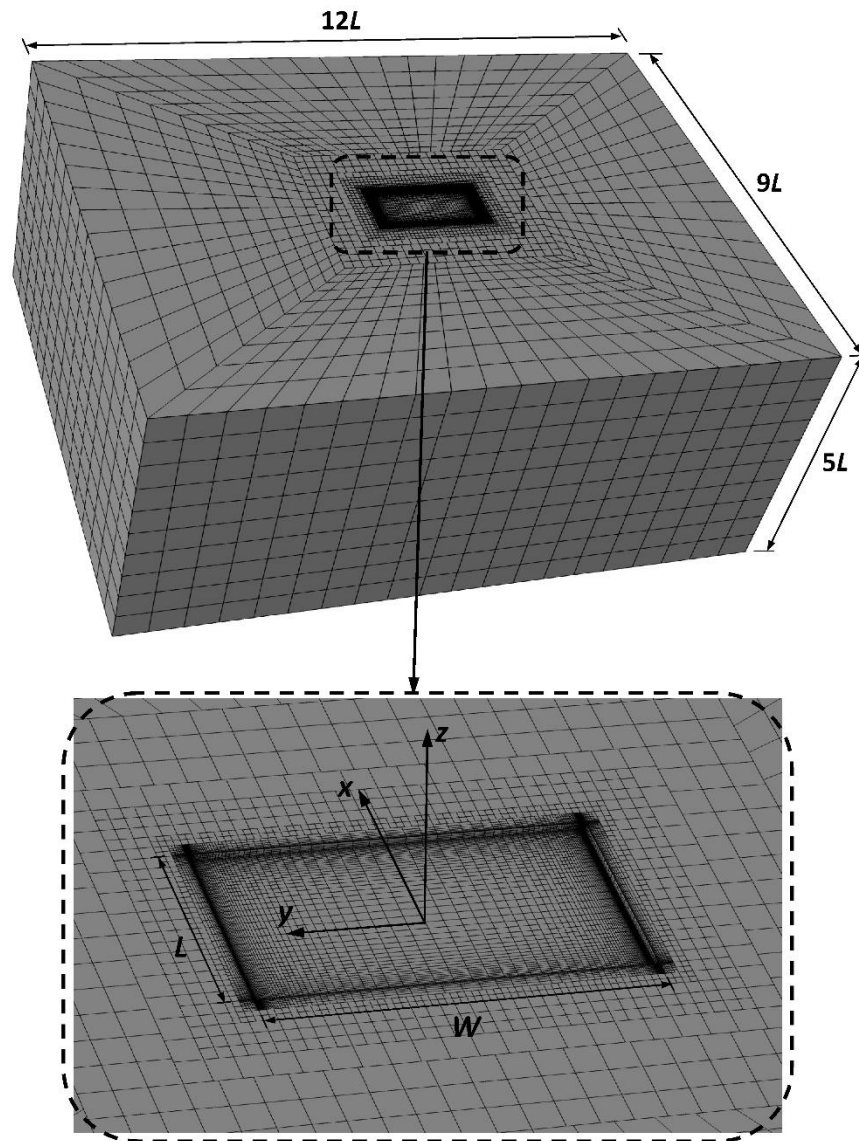


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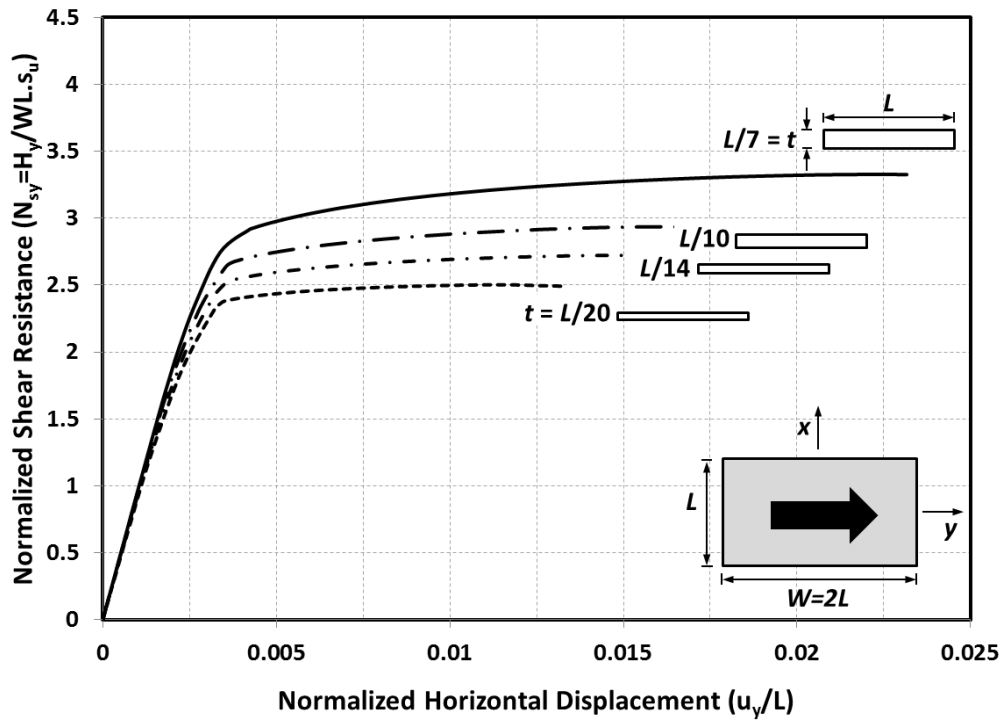
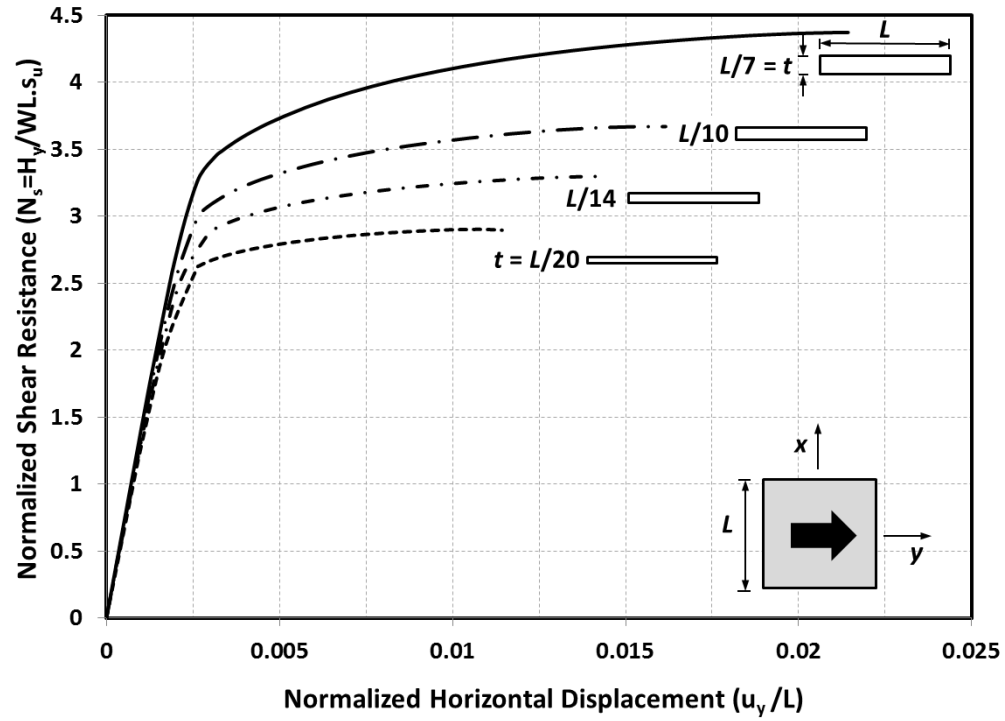


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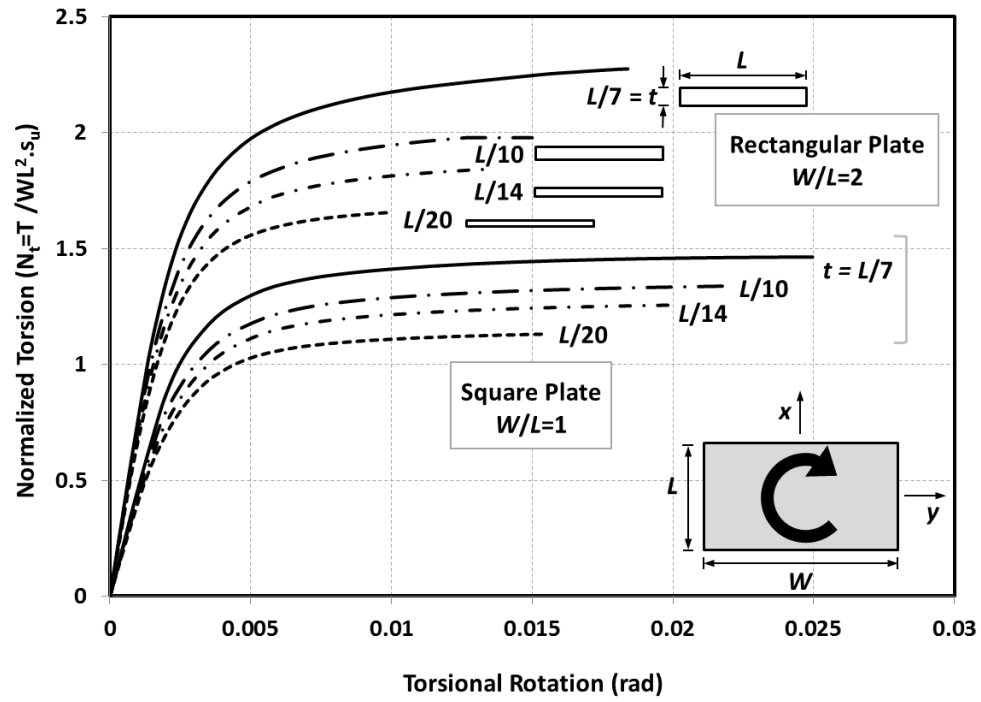


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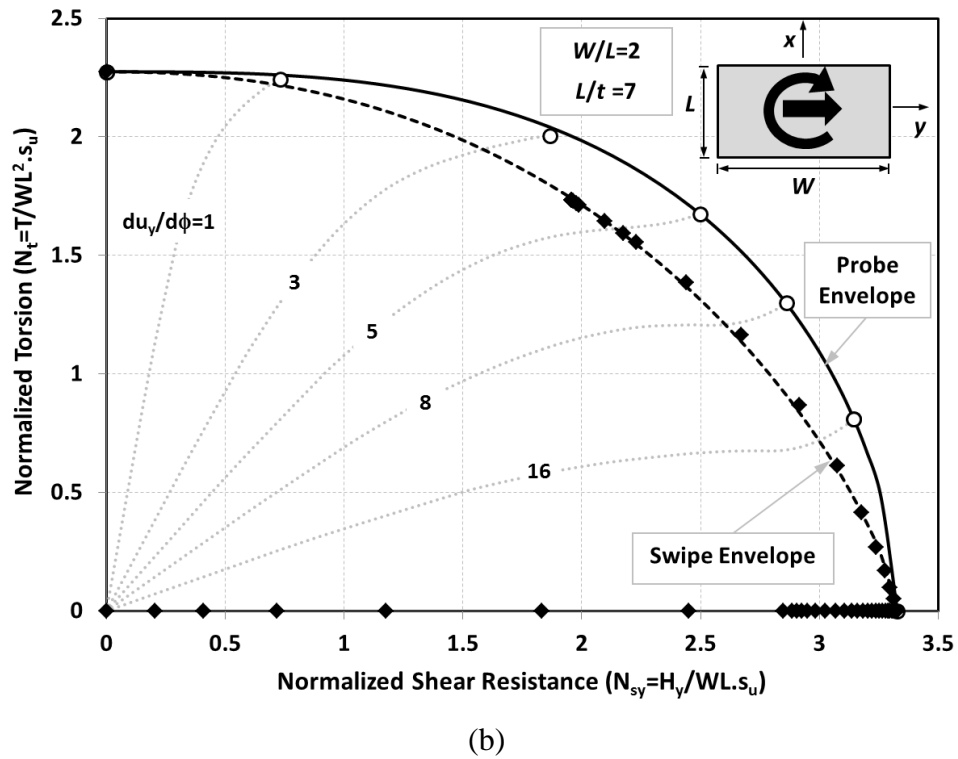
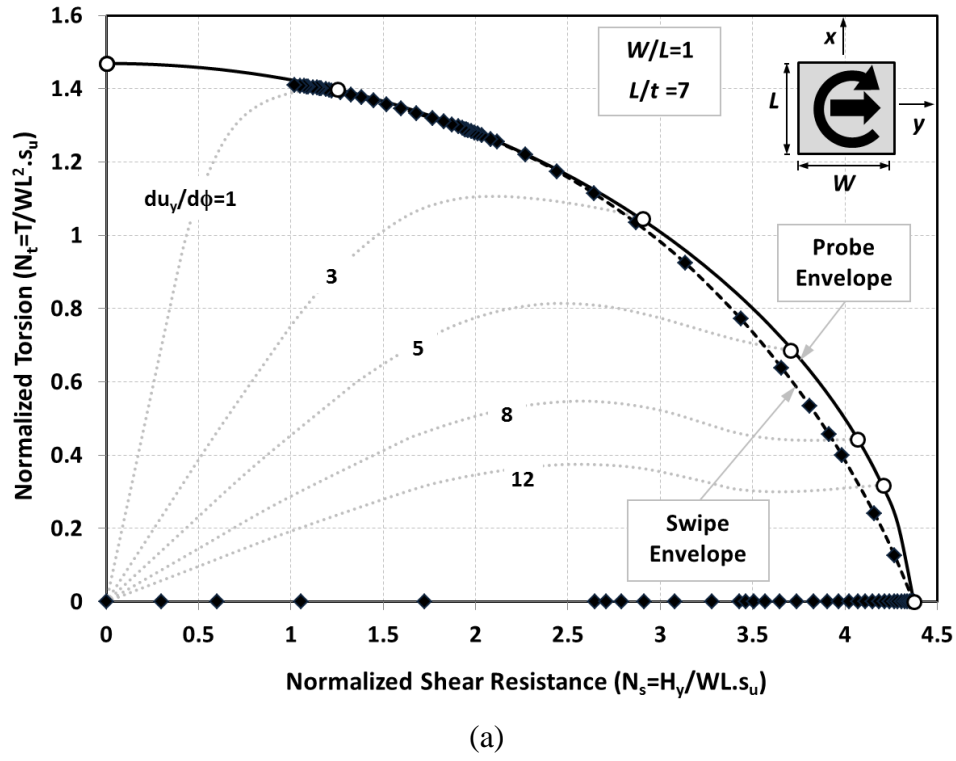


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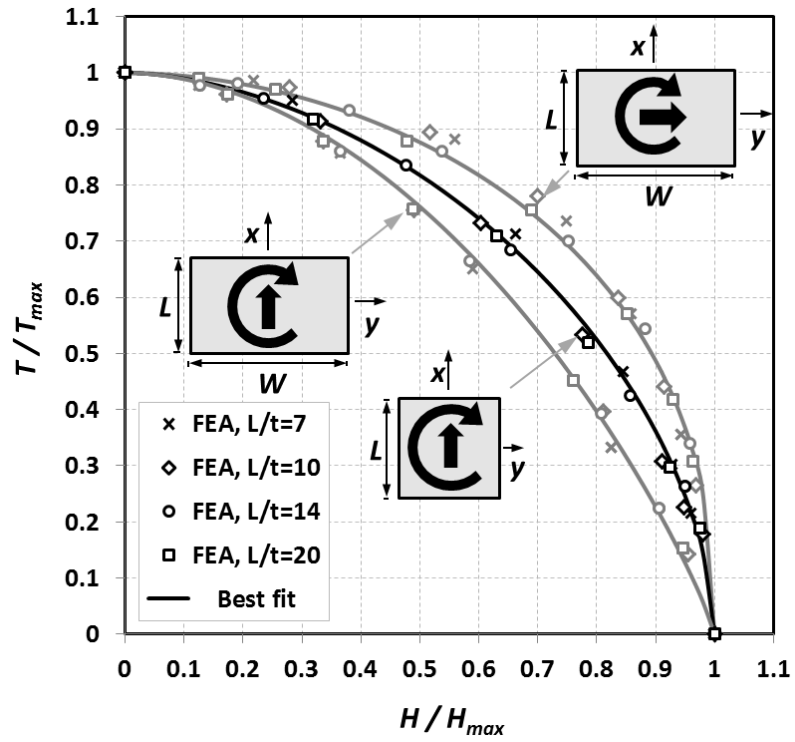


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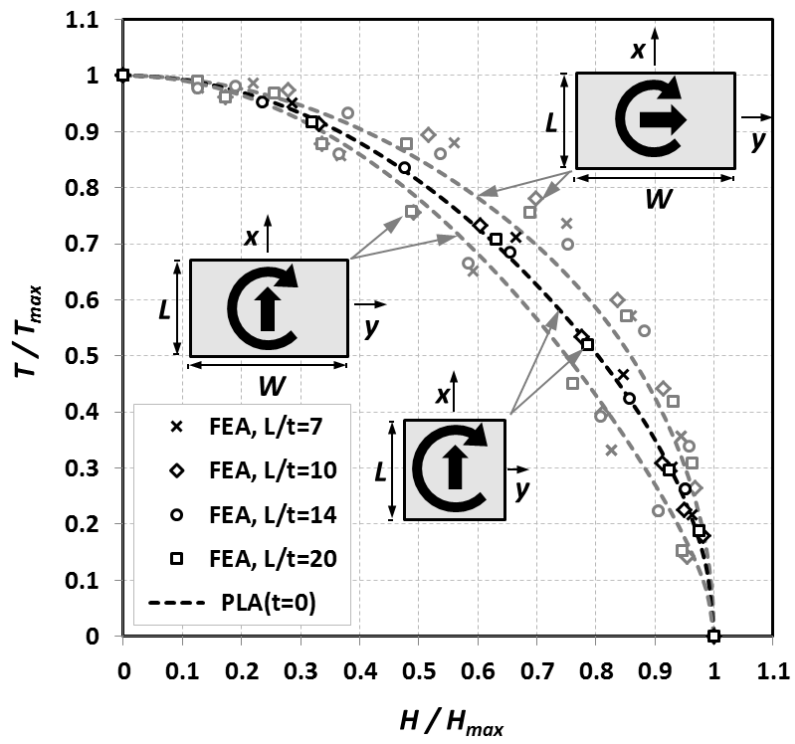
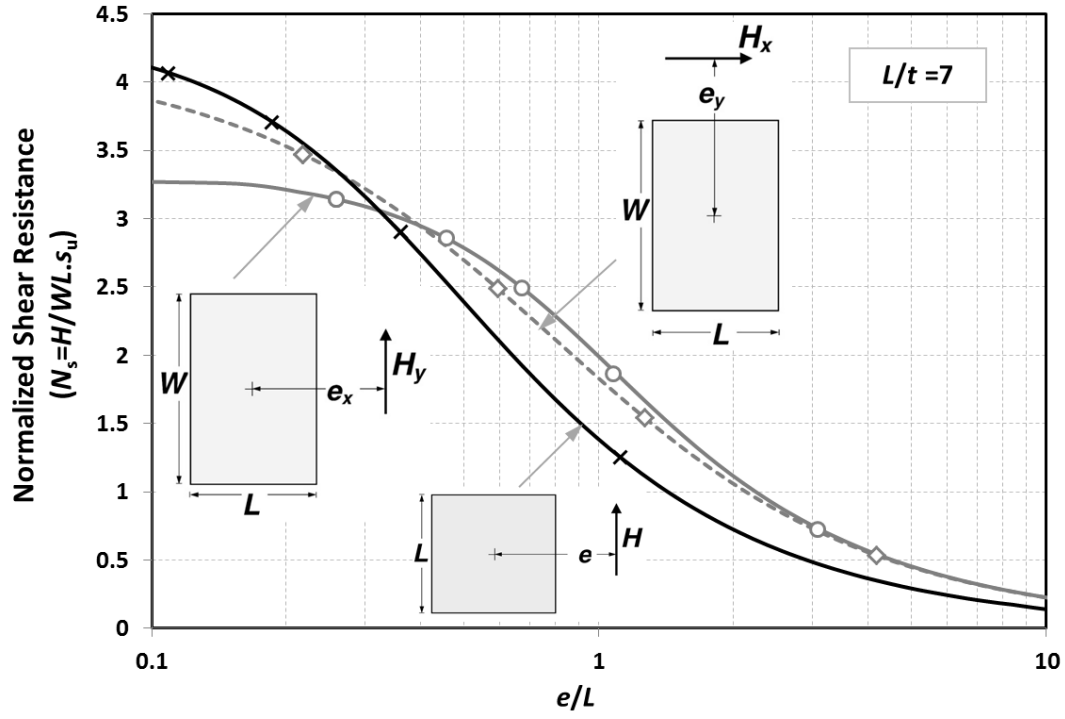
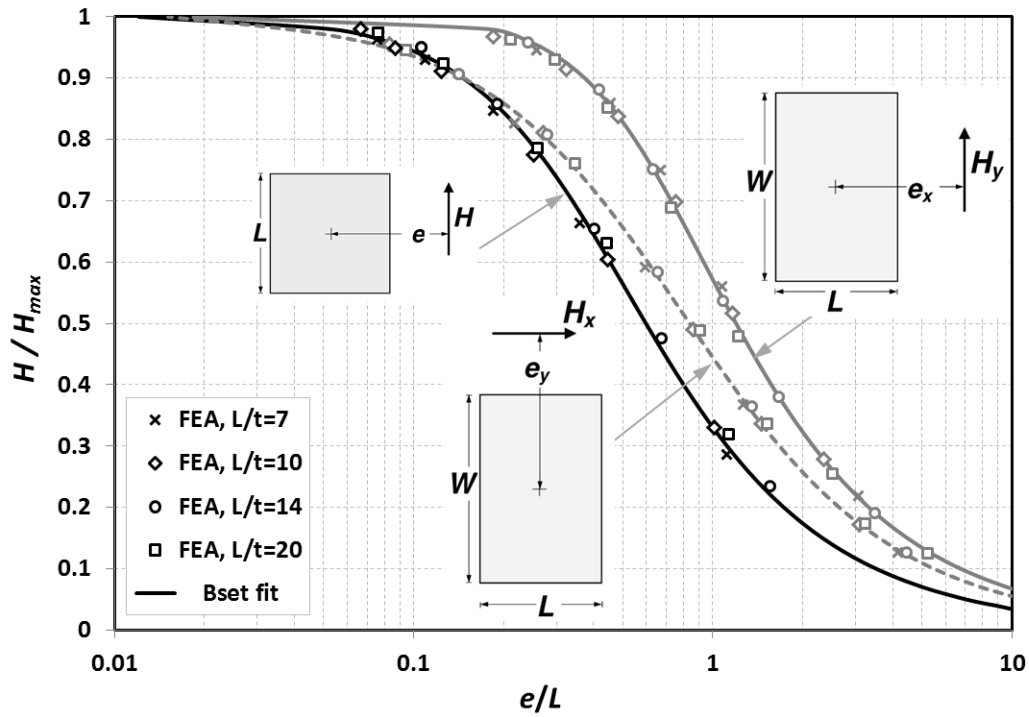


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(a)



(b)

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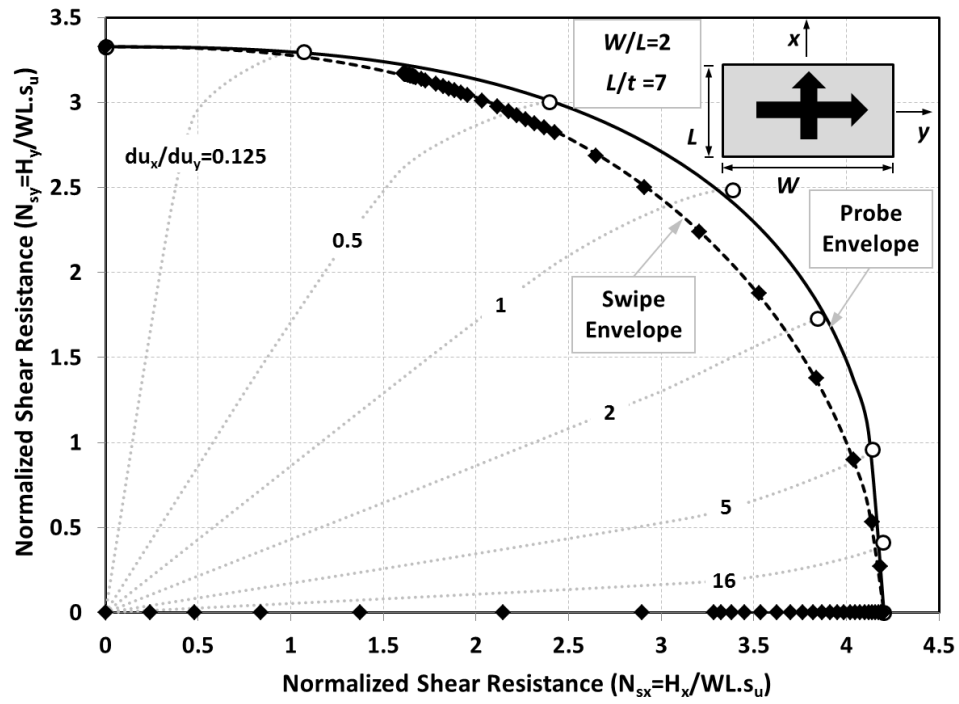


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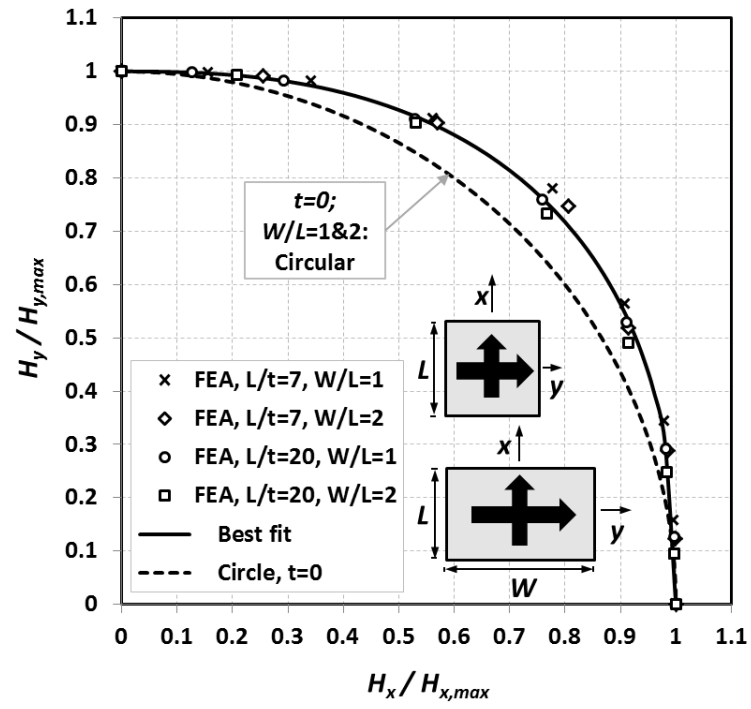


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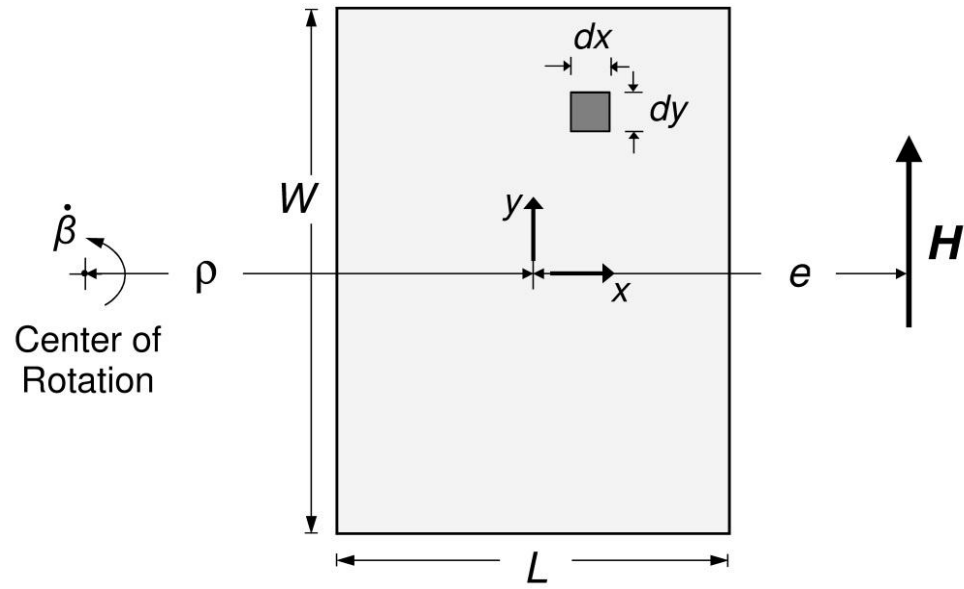


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